Proceedings of the
International Conference on Mechanical Engineering 2011
(ICME2011) 18-20 December 2011, Dhaka, Bangladesh

ICME 11-FL-027

EFFECTS OF GRAVITY AND FLUID PROPERTIES ON ISOTHERMAL HIGH SPEED TWO-LAYER FILAMEMT JET FLOW

Z. U. Ahmed and M. A. Wakil

Department of Mechanical Engineering, Khulna University of Engineering & Technology, Bangladesh.

ABSTRACT

In this paper, incompressible and isothermal two-layer Newtonian filament jet flow is investigated with layers of different rheological properties. The fiber with multi-layer filament jets has steadily been advanced in various applications, for example, in plastic fabrication process, inserting batteries, placing colors, burying recycled materials, coatings for controlling film surface properties etc. The present study focuses on the steady state of two-layer fiber spinning considering inertia, gravity and identifying non-uniform velocity of each layer across the fiber. Since the analytical solution is practically impossible, the governing equations are solved numerically as a nonlinear two-point boundary value problem. The effects of gravity and fluid properties (viscosity ratio, density ratio) are discussed. The velocity in each layer increases monotonically with position for any viscosity ratio, but the elongation rate depends strongly on the viscosity ratio. The overall velocity weakens in outer layer and strengthens in inner layer. The velocity in both layers decreases with the increasing density ratio, which suggests that elongational effects in both layers enhances. The velocity in both layers increases due to gravity, deviating gradually from the exponential growth to grow linearly with axial position.

Keywords: Fiber Spinning, Draw Ratio, Boundary Value Problem.

1. INTRODUCTION

In a typical fiber spinning operation, the molten polymer is extruded through a spinneret to create a thin cylinder. The molten fiber is normally stretched in length and drawn to reduce its diameter. The desired fiber diameter can be obtained by choosing the appropriate draw ratio (ratio of the take-up velocity to the extrusion velocity at the spinneret). In the actual process a bundle of many filaments are extruded and stretched together [1], hence, it is important to investigate multilayer fiber spinning process. The fiber with multi-layer filament jets has steadily been advanced in various applications, for example, in plastic fabrication process, inserting batteries, placing colors, burying recycled materials, coatings for controlling film surface properties etc [2].

One of the major instability and defects which limits the productivity in industrial operation is draw resonance. The draw resonance appears typically at high draw ratios. There exists a critical draw ratio, beyond which stable operation is impossible and draw resonance instability is observed. A multi-layer fiber spinning process is chosen due to the limited investigations exist in literature to analyze this draw resonance instability of Newtonian fluid.

Numerous studies have been conducted on fiber spinning of Newtonian and non-Newtonian fluid since early 1960’s when draw resonance was first introduced by Christensen [3] and Miller [4]. The previous experimental and theoretical studies on the fiber spinning process are mostly focused on single-layer flow. However, only relevant works related to the current study are described here. Kase and Matsuo [5] analyzed the dynamics of melt spinning by deriving a set of partial differential equations. Their steady-state solution showed fairly good agreement with the experimentally measured values of temperature and cross-sectional area. Later, assuming isothermal flow, Matovich and Pearson [6,7] and Ishihara and Kase [8] obtained analytical steady-state solution and stability analysis of fiber spinning, where most of the studies were without inertia, gravity, and surface tension. Experiments on Newtonian fluid were performed by Donnelly and Weinberger [9] and D’Andrea and Weinberger [10].

The inertia and gravity can, however, play an important role in stability of the real process. Considering viscous, inertia, gravity, and surface tension forces, respectively, Shah and Pearson [11] assessed their effects on stability. Their work showed that, in addition to the draw ratio, these three terms also play very important roles in fiber spinning stability. The effects of gravity on fiber spinning process, which have not been investigated in detail, especially about their compound effect, is one of the main objective in the present study. Gravity can also contribute great effect when its value is high enough.

Although extensive studies, either theoretical or experimental, have been conducted on single-layer fiber spinning process, the investigation of multilayer fiber spinning, even film casting, is very limited in literature. Park [12] performed the first theoretical analysis on two-layer film casting, where one layer was Newtonian and another was Upper-convected Maxwell fluid. Using simple constitutive models, Park investigated the effects of the interaction between two fluids with different rheological properties on the film thickness and stress profiles. Co et al. [13, 14] adopted a similar assumption and studied the multi-layer film casting of modified Giesekus fluids. Recently, the film casting of multi-layer flows of Newtonian fluid and non-Newtonian fluid was studied by Zhang et al. [15] and Lee et al. [16], respectively. Of closer relevance to the present study is the investigation of Suman and Tandon [17] who examined the three-layer flow stability of fiber drawing for both Newtonian and non-Newtonian fluids of constant densities in the absence of inertia, gravity and surface tension. They derived the governing equations as slender jet approximation, and did not treat three layers separately. They, instead, used a combined momentum balance equation. They imposed continuity of velocity vector on the layer interface, which predicts uniform velocities across the fiber cross section regardless the shearing in the layers. This prediction of continuous velocity at the interface was reported somewhat erroneous by Zhang et al. [15], who reported that there is bound to be a certain amount of shearing that increases with the viscosity ratio, hence the axial velocity cannot be (even approximately) uniform across the entire film. The details with the boundary condition, however, will be discussed in the next section. For the case of isothermal Newtonian drawing, Suman and Tandon [17] predicted the critical draw ratio for multi-layered fiber is 20.21, which is the same value that has been reported for the single layer fiber spinning. The fiber spinning processes of two- or multi-layer flows is essentially unexplored, especially in the presence of gravity.

So far, earlier studies pertain to fiber spinning, mostly focused on single-layer analysis of steady state and stability, except the study conducted by Suman and Tandon [17], who analyzed three-layer fiber drawing in the absence of inertia and gravity and with the assumption of uniform velocity across the entire fiber. The present study focuses on the steady state of two-layer Newtonian fiber spinning considering gravity and identifying non-uniform velocity of each layer across the fiber, which is clearly different from Suman and Tandon [17]. The major objective of this study is to investigate how the mechanics of the two-layer fiber spinning flow is affected by the properties of each layer. The influence of gravity, flow properties such as viscosity ratio, density ratio on the steady state flow will be investigated.

2. PROBLEM FORMULATION

Consider an axisymmetric, two-layer incompressible, Newtonian and isothermal fiber exiting from a die is drawn continuously and wrapped by the rotating cold roll at some distance down from the die exit. The distance between the spinneret and the chill roll is L. The jet is assumed to exit from two concentric circular tube type die, where one layer exits from the inner circular tube and the other layer, which completely surrounds the inner layer, exits from the thin annulus in between the circular tubes. The two layers, inner layer (layer 1) and outer layer (layer 2), are assumed immiscible, and attach each other immediately after the die exit. The problem is schematically described in figure 1. The radii of inner and outer laer are R^1 and R^2, respectively. The densities and viscosities of layer 1 and layer 2 are ρ^1 and ρ^2, and viscosities μ^1 and μ^2, respectively. The problem is formulated in the (r, θ, X) space with r, θ and X axis coinciding with the radial, azimuthal and axial directions. The corresponding velocities in the axis directions are (U_r, U_θ, U_x). The radii of two-layer fiber and mean velocities at the exit of die (X = 0) are, R_0^1 and R_0^2 and U_{x0}^1 and U_{x0}^2. The fiber drawing length (air gap) between the die and winder/chill roll is L. The die swell is also ignored, considering the variation of fiber radius is small. The flow is assumed to be dominant axial velocity that is uniform across each layer separately indicating the flow is predominantly elongational. The assumption is similar to all other previous studies on either single or multi-layer fiber spinning (even in film casting as well). The streamwise velocities of two layer are, thus, functions of axial direction X and time τ only. This independency of r causes a decoupling of U_r from U_x. Note, however, that U_r is not assumed to vanish.

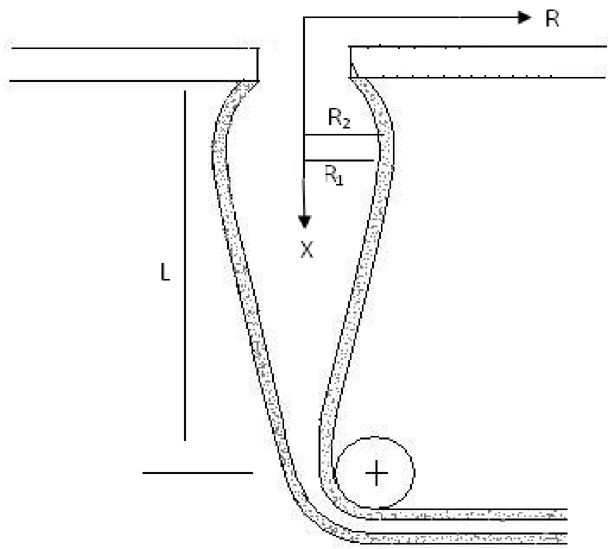

Fig 1. Schematic illustration of the two-layer fiber spinning process

It is convenient to cast the problem in dimensionless form. Thus, L will be taken as the reference length,

U_{x0}^1 the reference velocity, and R_0^1 the reference radius. The dimensionless variables of relevance to the problem are then defined as follows

$$x = \frac{X}{L}, \quad t = \frac{U_{x0}^1 \tau}{L}, \quad u_x^i = \frac{U_x^i}{U_{x0}^1}, \quad \delta^i = \frac{R^i}{R_0^1}. \qquad (12)$$

In this case six parameters emerge in the problem, namely the draw ratio, Dr, the velocity ratio, R_u, the density ratio, R_ρ, the viscosity ratio, R_μ, the Froude number, Fr, as well as Reynolds number, Re, which are introduced as

$$Dr = \frac{U_{xL}^1}{U_{x0}^1}, \quad R_r = \frac{R_0^2}{R_0^1}, \quad R_U = \frac{U_{x0}^2}{U_{x0}^1}, \quad R_\rho = \frac{\rho^2}{\rho^1}, \qquad (13)$$

$$R_\mu = \frac{\mu^1}{\mu^2}, \quad Fr = \frac{U_{x0}^{1^2}}{gL}, \quad Re = \frac{\rho^1 U_{x0}^1 L}{\mu^1}.$$

Upon using these dimensionless variables and parameters into the governing equations, the following non-dimensional governing equations are evolve,

$$\delta^{1^2}{}_{,t} + \left(\delta^{1^2} U_x^1\right)_{,x} = 0, \qquad (14)$$

$$\delta^{o^2}{}_{,t} + \left(\delta^{o^2} U_x^2\right)_{,x} = 0 \qquad (15)$$

$$\frac{Re}{3}\left[\left(\delta^{1^2} + R_\rho R_\mu \delta^{o^2}\right) u_{x,t}^1 + \left(\delta^{1^2} + R_\rho R_\mu^2 \delta^{o^2}\right) u_x^1 u_{x,x}^1 + R_\rho R_\mu \delta^{o^2}\left(R_U - R_\mu\right) u_{x,x}^1\right] = \left[\left(\delta^{1^2} + \delta^{o^2}\right) u_{x,x}^1\right]_{,x} + \frac{Re}{3Fr}\left(\delta^{1^2} + R_\rho \delta^{o^2}\right), \qquad (16)$$

$$u_x^2 = R_\mu\left(u_x^1 - 1\right) + R_U, \qquad (17)$$

where $\delta^{o^2} = \delta^{2^2} - \delta^{1^2}$ in equations (15) and (16).

The boundary conditions are reduced to

$$u_x^1(x=0,t) = 1, \quad u_x^1(x=1,t) = Dr, \quad \delta^1(x=0,t) = 1, \quad \delta^2(x=0,t) = R_r. \qquad (18)$$

The governing equations (14)-(17) and boundary condition (18) constitute a two-point boundary value problem.

The steady state radii can readily be expressed in terms of velocities from equation (14) and (15) upon using condition (18) as follows

$$\delta^{1s} = \frac{1}{\sqrt{u_x^{1s}}}, \qquad \delta^{os} = \sqrt{\frac{R_r{}^2 R_U}{R_\mu\left(u_x^{1s} - 1\right) + R_U}}. \qquad (19)$$

Upon substituting the expressions from (19) for the steady state radii and from (17) for the steady state velocity of layer b into (16), the equation for the velocity of layer a is obtained,

$$\frac{Re}{3}\left[\left(\delta^{1s^2} + R_\rho R_\mu^2 \delta^{os^2}\right) u_x^{1s} u_{x,x}^{1s} + R_\rho R_\mu \delta^{os^2}\left(R_U - R_\mu\right) u_{x,x}^{1s}\right] = \left[\left(\delta^{1s^2} + \delta^{os^2}\right) u_{x,x}^1\right]_{,x} + \frac{Re}{3Fr}\left(\delta^{1s^2} + R_\rho \delta^{os^2}\right) \qquad (20)$$

with the boundary conditions

$$u_x^{1s}(x=0) = 1, \qquad u_x^{1s}(x=1) = Dr. \qquad (21)$$

Due to the unavailability of analytical solution, the equation (20) with conditions (21) is solved numerically by two-point boundary value problem using MATLAB. The coupled governing equation is recast as the system of first order equation. The nonlinear system is solved by using function 'bvp4c'. Numerical solution is obtained by solving a global system of algebraic equations stemming from the boundary conditions. The mesh is chosen adaptively to make the local error in the tolerance limit, which is chosen 1e-6 for all computations.

3. RESULTS AND DISCUSSION

In this section, the formulation and the numerical solution are applied to the two-layer film fiber spinning flow as schematically illustrated in Figure 1. The velocity profiles and the film radii distributions for each layer will be determined. The influence of gravity and the properties of the fluid (viscosity and density ratios) on the flow are investigated. All the results are given in terms of dimensionless quantities.

The effect of viscosity on the velocities and the fiber radii is explored by examining the two-layer fiber spinning flow for different values of the viscosity ratio, $R_\mu \in [0.25, 4]$. The remaining parameters are fixed at D_R =15, Fr =Re=1, Rρ =0.5, Ru =1 and Rr =1.2. The flow response is delineated in Figures 2.

It is observed that the velocity in each layer increases monotonically with X regardless of Rμ, but the elongation rate depends strongly on Rμ, and is different for each layer. As Rμ increases, the overall velocity slackens in layer a and intensifies in layer b. The velocity sharply increase at the take up point in layer b which of course suggests that the flow in layer a debilitates as the layer becomes relatively more viscous, leaving layer b to flow as fast as seen in Figure 2. It is expected that as Rμ increase further, the flow activity in layer a becomes increasingly limited to the region near the take up point, with the layer behaving more like a solid almost everywhere except at the tip where considerable stretching occurs. The overall radius of layer a increases with Rμ, while the overall radius of layer b decreases with viscosity ratio (Figure 2).

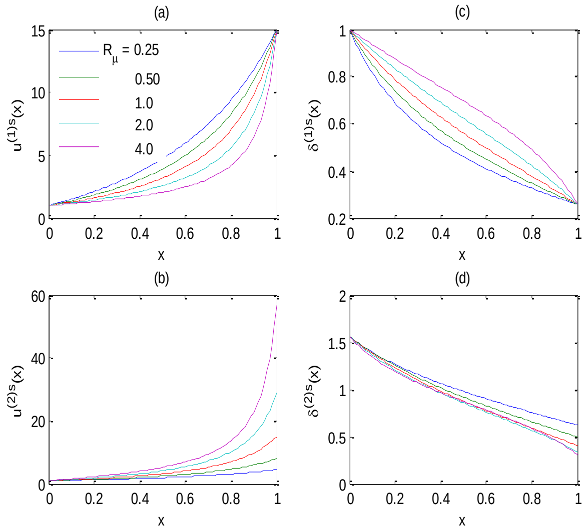

Fig 2. Influence of viscosity ratio on the velocities and fiber radii, for $R_\mu \in [0.25, 4]$, $Re = 1$, $R_\rho = 0.5$, $R_r = 1.2$, $R_u = 1$, Fr = 0.1, Dr = 15

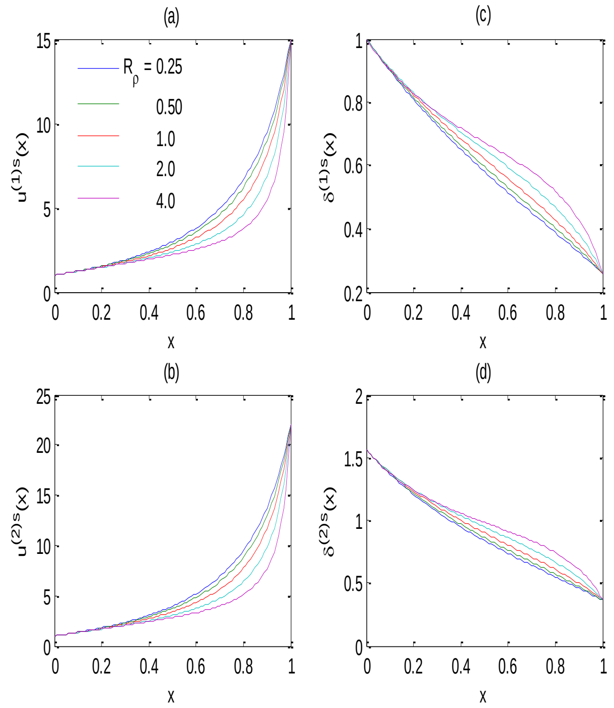

Fig 3. Influence of density ratio on the velocities and fiber radii, for $R_\rho \in [0.25, 4]$, $Re = 1$, $R_\mu = 1.5$, $R_r = 1.2$, $R_u = 1$, Fr = 0.1, Dr = 15

The influence of the density ratio on the flow is investigated by varying Rρ from 0.25 to 4. In this case Dr =15, Rμ=1.5, and Re=Fr =Ru =1. Rr =1.2. The flow response is depicted in Figures 3. Only results with new features are reported in these figures. It is interesting to note, at the outset, that if all the fluid parameters are equal in layers a and b, the resulting velocity and radius distributions would be the same as those corresponding to single-layer film spinning even when the densities in the two layers are not equal. The velocity in both layers decreases with the increasing density ratio, which suggests that elongational effects in both layers enhances. The flow elongation is strong enough to halt the decrease of the film thickness in layer a with X. The velocity and radius at the take-up point in each layer are not affected by Rρ.

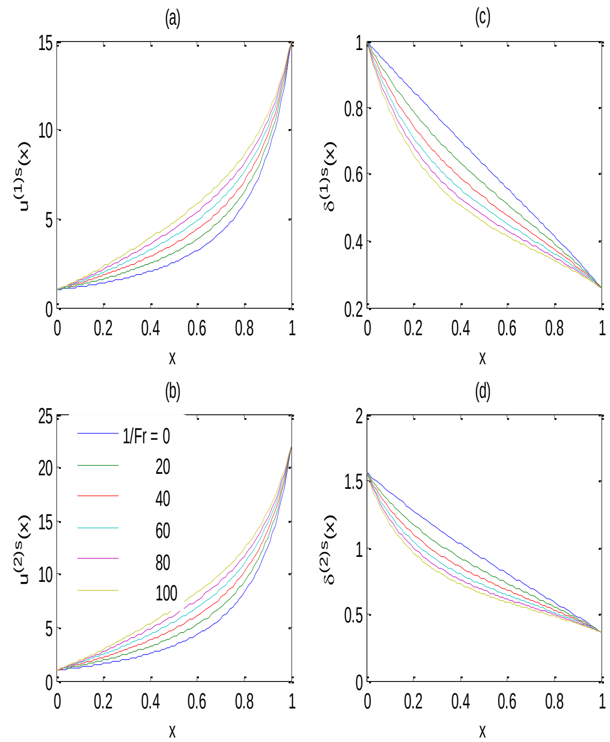

Fig 4. Influence of gravity on the velocities and fiber radii, for $G \in [0,100]$, $Re = 1$, $R_\rho = 0.5$, $R_\mu = 1.5$, $R_r = 1.2$, $R_u = 1$, Dr = 15

The effect of gravity is examined by varying the Fround number with G in the range 0 to 100. The remaining parameters are fixed at Dr =15, Re=1, Rμ =1.5, Rρ =0:5, Rr =1.2 and Ru =1. The overall velocity in both layers increases due to gravity, deviating gradually from the exponential growth to grow linearly with X as Fr decreases. Simultaneously, the film thickness begins to decrease rather sharply near the die exit as gravity increases as portrayed in figure 4.

4. CONCLUSION

In this paper, steady, incompressible and isothermal two-layer Newtonian filament jet flow is investigated with layers of different rheological properties. Due to the unavailability of analytical solution, the governing equation is solved numerically using two-point boundary value problem. The effects of gravity and flow properties on the steady state two-layer filament jet are observed.

The velocity in each layer increases monotonically with X regardless of Rμ, but the elongation rate depends strongly on Rμ. The overall velocity weakens in layer a and strengthens in layer b with the increase of Rμ. The overall radius of layer a increases with Rμ, while the overall radius of layer b decreases with viscosity ratio. The velocity in both layers decreases with the increasing density ratio, which suggests that elongational effects in both layers enhances. The velocity and radius at the take-up point in each layer are not affected by Rρ. The overall velocity in both layers increases due to gravity, deviating gradually from the exponential growth to grow linearly with axial position.

5. REFERENCES

1. J.A. Dantzig and C.L. Tucker, *Modeling in Materials Processing*. Cambridge University Press, Cambridge, 2001
2. J. S. Lee, D. M. Shin, H. W. Jung, J. C. Hyun, Stability analysis of a three-layer film casting process, Korea-Australia Rheology Journal, 19, No. 1, 2007, 27-33
3. R.E. Christensen, Extrusion coating of polypropylene, *S.P.E.J.*,18 (1962) pp.751
4. J.C. Miller, Swelling behavior in extrusion, *S.P.E.Trans.*, 3 (1963) pp. 134
5. S. Kase and T. Matsuo, Studies on Melt Spinning I. Fundamental Equations on the Dynamics of Melt Spinning, *J. Polym. Sci. Part A.* **3**, 2541 (1965)
6. M. A. Matovich and J. R. A. Pearson, Spinning a Molten Thread Line: Steady State Isothermal Viscous Flows, *Ind. Eng. Chem. Fund.* **8**, 512 (1969a)
7. J. R. A. Pearson and M. A. Matovich, Spinning a Molten Thread Line: Stability, Ind. Eng. Chem. Fund. **8**, 605 (1969b)
8. H. Ishihara and S. Kase, Studies on Melt Spinning V. Draw Resonance as a Limit Cycle, *J. Appl. Polym. Sci.* **19**, 557 (1975)
9. G. J. Donnelly and C. B. Weinberger, Stability of Isothermal Fiber Spinning of a Newtonian Fluid, *Ind. Eng. Chem. Fund.* **14**, 334 (1975)
10. R. G. D'Andrea and C. B. Weinberger, Effects of Surface Tension and Gravity Forces in Determining The Stability of Isothermal Fiber Spinning, *AICHE J.* **22**, 923 (1976)
11. Y. T. Shah and J. R. A. Pearson, On the Stability of Non-isothermal Fiber Spinning – General Case, *Ind. Eng. Chem. Fund.* **11**, 150 (1972)
12. Park C-W. A study on bicomponent two-layer slot cast coextrusion. Polymer Engineering and Science 1991; 31:197–203.
13. J.A. Dantzig and C.L. Tucker, *Modeling in Materials Processing*. Cambridge University Press, Cambridge, 2001
14. R.E. Christensen, Extrusion coating of polypropylene, *S.P.E.J.*,18 (1962) pp.751
15. J. Zhang, R. E. Khayat, W. Wang, Stability of high-speed two-layer film casting of Newtonian fluids, Int. J. Numer. Meth. Fluids, 2006, 52, 31-61
16. J. S. Lee, D. M. Shin, H. W. Jung, J. C. Hyun, Stability analysis of a three-layer film casting process, Korea-Australia Rheology Journal, 19, No. 1, 2007, 27-33
17. B. Suman, P. Tandon, Fluid flow stability analysis of multilayer fiber drawing, Chem. Engg. Sc., 65, 2010, 5537-5549.

8. MAILING ADDRESS

Z. U. Ahmed
Department of Mechanical Engineering
Khulna University of Engineering & Technology
Khulna, Bangladesh
Email ID: zuasohel@hotmail.com